基于 MATLAB 的水力裂缝扩展数值模拟技术

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摘要：水力裂缝扩展数值模拟是水力压裂设计的重要内容，可以用来评价压裂效果，降低施工风险，为节省计算时间目前主要采用二维模型或者拟三维模型。现今比较成熟的 GOHFER 商业软件，以差分方程代替微分方程，误差逐层逸散不断扩大，差分格式稳定性难以保证，为此，采用 MATLAB 编程语言编制模拟程序，实现有限元数值分析对水力裂缝扩展真三维模型的求解，其适应性强，边界条件自动满足，且以高阶多项式逼近真实值，误差较小，精度更高。同时，采用复化辛普森数值积分代替 MATLAB 编程语言中的符号积分，节省了 61.9% ~73.5% 的运算时间。

关键词：水力压裂 有限元 真三维模型 裂缝扩展 数值模拟
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水力压裂技术在低渗透油藏尤其是页岩油藏[1]开发中应用越来越普遍，裂缝扩展数值模拟[2]是压裂设计的重要内容。

目前，国外商业软件 GOHFER 主要以二维有限差分划分网格，计算岩石力学参数，精度较差，而有限元通过单元特性矩阵组装整体特性矩阵，以单元节点处的场变量构建多项式，计算精度较准确。MATLAB 编程语言允许用数学形式的语言编写程序，比 Visual Basic 和 Fortran 及 C＋＋更加接近书写计算公式的思维，编程效率高，扩充能力强，拥有丰富的库函数，语句简单，内涵丰富，在矩阵和数组运算方面更加高效方便，可视化操作和图形处理功能强大，并且数值积分的使用又大大降低了运算时间，使得在 MATLAB 中实现水力裂缝扩展真三维模拟简单易行且更切合实际。

1　水力压裂真三维模型

在实际压裂中，由于射孔孔眼附近应力集中[3]、端部脱砂[4]和砂砾岩压裂[5]等问题，导致水力裂缝扩展形态复杂，出现"T"形扩展和滑移[6]等现象，对水力裂缝模拟提出了更高的要求，应该更切合实际地反映裂缝动态扩展过程。

1.1　裂缝宽度和净压力模型

Palmer 等[7]认为压裂液属于牛顿流体，油层裂缝扩展真三维模型主要由压裂液连续性方程、幂律流体的流动方程和弹塑性方程 3 部分构成。

以射孔中心为原点，裂缝延伸方向为 x 轴，井筒垂直方向为 y 轴，建立坐标系，则压裂液连续性方程为

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho v_x w)}{\partial x}+\frac{\partial(\rho v_y w)}{\partial y}=-q_1\rho_f \tag{1}$$

其中

$$q_1=\frac{2c_1}{\sqrt{t-\tau(x,y)}} \tag{2}$$

式中：ρ 为携砂液密度，kg/m^3；w 为裂缝宽度，m；t 为泵注时间，min；v_x 为流体沿裂缝延伸方向的流速，m/min；x 为地层中任意点到井筒垂直轴线的距离，m；v_y 为流体沿井筒方向的流速，m/min；y 为地层中任意点到射孔中心的距离，m；q_1 为滤失量，m/min；ρ_f 为压裂液基液密度，kg/m^3；c_1 为滤失系数，m/min$^{0.5}$；τ 为压裂液到达水力裂缝壁面上任意点的时间，min。

幂律流体的流动方程为

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$$\begin{cases} wv_x = -\dfrac{n}{2n+1}K^{-\frac{1}{n}} \times \\ \left[\left(\dfrac{\partial p}{\partial x}\right)^2 + \left(\dfrac{\partial p}{\partial y}\right)^2\right]^{\frac{1-n}{2n}} \times \dfrac{w^{\frac{2n+1}{n}}}{2^{\frac{n+1}{n}}} \times \dfrac{\partial p}{\partial x} \\ wv_y = -\dfrac{n}{2n+1}K^{-\frac{1}{n}} \times \\ \left[\left(\dfrac{\partial p}{\partial x}\right)^2 + \left(\dfrac{\partial p}{\partial y}\right)^2\right]^{\frac{1-n}{2n}} \times \dfrac{w^{\frac{2n+1}{n}}}{2^{\frac{n+1}{n}}} \times \dfrac{\partial p}{\partial y} \end{cases} \tag{3}$$

式中：n 为流体的流变系数；K 为流体的稠度系数，mPa·s^n；p 为裂缝内净压力，MPa。

以射孔中心为原点，井筒竖直轴线为 y' 轴，裂缝延伸方向为 x' 轴，建立与 (x,y) 坐标系平行的 (x',y') 坐标系，建立的弹塑性方程[8-9]为

$$-p_L(x,y) + \sigma(x,y) = \frac{G}{4\pi(1-\nu)} \times \iint_\Omega \left[\frac{\partial}{\partial x}\left(\frac{1}{r}\right) \times \frac{\partial w}{\partial x'} + \frac{\partial}{\partial y}\left(\frac{1}{r}\right) \times \frac{\partial w}{\partial y'}\right] \mathrm{d}x'\mathrm{d}y' \tag{4}$$

式中：p_L 为缝壁面压力，MPa；σ 为地层最小主应力，MPa；G 为地层岩石剪切模量，MPa；ν 为泊松比；Ω 为二维整体裂缝内部积分空间；r 为 Ω 内部二重积分任意点至二维裂缝面内部任意点的距离，m；x' 为 Ω 内部二重积分点到井筒轴线的距离，m；y' 为 Ω 内部二重积分点到射孔中心的距离，m。

1.2　铺砂浓度模型

支撑剂运移方程为

$$\frac{\partial(c\rho_p w)}{\partial t} + \nabla\cdot(c\rho_p w\nu) = 0 \tag{5}$$

式中：c 为压裂液中支撑剂铺砂浓度，kg/m^3；ρ_p 为压裂液支撑剂携砂液密度，kg/m^3。

由于携砂液质量为压裂液质量和支撑剂质量之和，即

$$\rho V = \rho_f V_f + \rho_p V_p \tag{6}$$

式中：V 为携砂液体积，m^3；V_f 为压裂液基液体积，m^3；V_p 为支撑剂体积，m^3。

因 $V_f = V - V_p$，$c = V_p/V$，$\rho = c\rho_p + (1-c)\rho_f$，将其代入式(1)得

$$-q_l\rho_f = \frac{\partial}{\partial t}\left\{[c\rho_p + (1-c)\rho_f]w\right\} + \nabla\cdot\left\{[c\rho_p + (1-c)\rho_f]w\nu\right\} \tag{7}$$

又因为 ρ_f 为一定值，通过式(5)和式(7)可得

$$q_l = wv_x\frac{\partial c}{\partial x} + wv_y\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial t} - (1-c)\left[\frac{\partial w}{\partial t} + \frac{\partial(wv_x)}{\partial x} + \frac{\partial(wv_y)}{\partial y}\right] \tag{8}$$

2　有限元模型的建立

2.1　裂缝宽度和净压力有限元模型

有限元模型首先用单元节点处的净压力和裂缝宽度来构建多项式，再以此多项式来表示缝壁面整体净压力和裂缝宽度[10]的变化，即

$$p(x,y) = [\phi(x,y)][p] \tag{9}$$

$$w(x,y) = [\phi(x,y)][w] \tag{10}$$

其中

$$[\phi(x,y)] = [\phi_i(x,y)\ \phi_j(x,y)\ \phi_k(x,y)] \tag{11}$$

$$[p] = [p_i, p_j, p_k]^{\mathrm{T}} \tag{12}$$

$$[w] = [w_i, w_j, w_k]^{\mathrm{T}} \tag{13}$$

式中：$p(x,y)$ 为以多项式形式表示的单元净压力分布函数；$w(x,y)$ 为以多项式形式表示的单元裂缝宽度分布函数；i，j 和 k 分别为三角形网格的 3 个不同节点；ϕ_i，ϕ_j，ϕ_k 为有限元基函数；p_i，p_j，p_k 为单元节点处的净压力，MPa；w_i，w_j，w_k 为单元节点裂缝宽度，m。

净压力有限元模型用矩阵表示为

$$[K_{pij}]_{3\times3}[p] = -[P_l]_{3\times1} - [P_t]_{3\times1} + [P_q]_{3\times1} \tag{14}$$

其中

$$K_{pij} = \int_\Omega \frac{nK^{-\frac{1}{n}}}{2n+1} \times \frac{w^{\frac{2n+1}{n}}}{2^{\frac{n+1}{n}}} \times \left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right]^{\frac{1-n}{2n}} \times \left(\frac{\partial\phi_i}{\partial x} \times \frac{\partial\phi_j}{\partial x} + \frac{\partial\phi_i}{\partial y} \times \frac{\partial\phi_j}{\partial y}\right)\rho\,\mathrm{d}x\mathrm{d}y \tag{15}$$

$$P_{li} = \int_\Omega \frac{2c_1}{\sqrt{t-\tau}}\phi_i\rho_f\,\mathrm{d}x\mathrm{d}y \tag{16}$$

$$P_{ti} = \int_\Omega \frac{\partial w}{\partial t}\rho\phi_i\,\mathrm{d}x\mathrm{d}y \tag{17}$$

$$P_{qi} = \int_{\partial\Omega} q\rho\phi_i\,\mathrm{d}s \tag{18}$$

式中：$\partial\Omega$ 为积分空间边界；q 为泵注速率，m^3/min；ds 为积分空间边界 $\partial\Omega$ 上的积分微元。

裂缝宽度有限元模型矩阵形式为

$$[K_{wij}]_{3\times3}[w] = [P_w]_{3\times1} \tag{19}$$

其中

$$K_{wij} = \frac{G}{4\pi(1-\nu)} \times \iint_{\Omega\Omega} \frac{1}{r}\left(\frac{\partial\phi_i}{\partial x} \times \frac{\partial\phi_j}{\partial x'} + \frac{\partial\phi_i}{\partial y} \times \frac{\partial\phi_j}{\partial y'}\right)\rho\,\mathrm{d}x'\mathrm{d}y'\mathrm{d}x\mathrm{d}y \tag{20}$$

$$P_{wi} = \int_{\Omega} \frac{\partial w}{\partial t} \rho \phi_i \mathrm{d}x\mathrm{d}y \quad (21)$$

用 $[K_p]$ 组装整体特性矩阵，$[P_l]$，$[P_t]$，$[P_q]$ 组装整体特性列阵，构建整体运算方程组，在 MATLAB 中求解该方程组，即可得到网格节点处的净压力。与此类似，通过 $[K_w]$ 和 $[P_w]$ 可以求取节点处的裂缝宽度。

2.2 铺砂浓度有限元模型

支撑剂铺砂浓度的求取采用与裂缝宽度和净压力模型相同的方法和思路。同样采用三角形网格节点，支撑剂铺砂浓度表示为

$$[c]_{3\times1} = [c_i, c_j, c_k]^{\mathrm{T}} \quad (22)$$

式中：c_i，c_j，c_k 为单元节点处支撑剂铺砂浓度，$\mathrm{kg/m^3}$。

通过式(8)建立泛函，推导的支撑剂铺砂浓度有限元模型简化后为

$$[A_{ij}]_{3\times3} \frac{\mathrm{d}[c]_{3\times1}}{\mathrm{d}t} + [B_{ij}]_{3\times3}[c]_{3\times1} = [P_c]_{3\times1} \quad (23)$$

其中

$$A_{ij} = \int_{\Omega} w(x,y)\phi_i(x,y)\phi_j(x,y)\mathrm{d}x\mathrm{d}y \quad (24)$$

$$B_{ij} = \int_{\Omega} \frac{n}{2n+1} K^{-\frac{1}{n}} \left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right]^{\frac{1-n}{2n}} \times \frac{w^{\frac{2n+1}{n}}}{2^{\frac{n+1}{n}}} \left(\frac{\partial p}{\partial x} \times \frac{\partial \phi_i}{\partial x} + \frac{\partial p}{\partial y} \times \frac{\partial \phi_i}{\partial y}\right)\phi_j \mathrm{d}x\mathrm{d}y \quad (25)$$

$$P_{ci} = \iint_{\Omega} \Big\{\left(q_l + \frac{\partial w}{\partial t}\right)\phi_i + \frac{n}{2n+1} K^{-\frac{1}{n}} \left[\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right]^{\frac{1-n}{2n}} \times \frac{w^{\frac{2n+1}{n}}}{2^{\frac{n+1}{n}}} \left(\frac{\partial p}{\partial x} \times \frac{\partial \phi_i}{\partial x} + \frac{\partial p}{\partial y} \times \frac{\partial \phi_i}{\partial y}\right)\Big\}\mathrm{d}x\mathrm{d}y \quad (26)$$

3 网格的划分与推进

网格划分 MATLAB 中自定义 delaunay 三角剖分函数，自动划分三角形网格，利用 trianmeshxy 子程序判断出符合条件的点集，并将其坐标存储在 2 个特定数组中，然后用 MATLAB 中的 delaunay 函数划分 DT 三角形网格，用自定义子程序 tripolt 将网格勾画出来，网格尺寸定义越小，网格划分越为精密，计算精度就越准确。

网格推进 裂缝总是沿着岩石最容易破裂的方向推进，根据断裂力学理论，当地层某处的应力强度因子大于该处的岩石断裂韧性时，该处岩石发生破裂，可用自定义子程序 SkeletonMovement 实现网格的推进。

4 复化辛普森数值积分求解

考虑到计算精度的问题，有限元网格不能划分过粗，理论上划分越细越好，计算将更为准确。但是由于裂缝扩展模型复杂(2 次二重积分)和 MATLAB 符号运算耗时耗内存，导致模拟时间延长，其中一项改进的方法就是用复化辛普森数值积分代替二重积分。软件中共有 7 个子程序采用该数值积分方法。从子程序分别采用符号积分和数值积分运算一次所需要的时间(表 1)可见，节省运算时间最多的是 InitiTriP.m，该程序用来进行净压力整体特性矩阵的初始化，采用数值积分后节省了 73.5% 的运算时间，节省时间最少的 Cacufw.m 也节省了 61.9%。

表 1 子程序运算一次所需时间 s

子程序	符号积分	数值积分
Cacufw.m	2.1	0.8
Cacufq.m	2.0	0.7
InitiTriP.m	3.4	0.9
InitiTriW.m	3.6	1.1
LiTriC.m	3.4	1.2
LiTrirP.m	3.7	1.4
LiTrW.m	3.8	1.4

5 现场应用

以胜利油区滨 660－斜 24 压裂井为例，该井井深为 3 004 m，改造油层段垂直井深为 2 801～2 824 m，油层温度为 110 ℃，中部射开 6 m，分别采用 MATLAB 真三维有限元模拟(图 1a)和 GOHFER(图 1b)模拟。同时在压裂过程中采用微破裂影像技术监测裂缝。将监测结果(图 1c)作为实际裂缝尺寸对比基准，对比分析计算与监测裂缝尺寸参数(表 2)可知，MATLAB 真三维有限元模拟取得的缝高和支撑缝长数据与监测结果更为接近。在 GOHFER 差分模拟中显示的油层上部滤失而下部突进的现象并没有在 MATLAB 模拟中体现出来。原因就在于真三维模型对裂缝形态描述更切合实际，有限元分析也比有限差分更为精确。

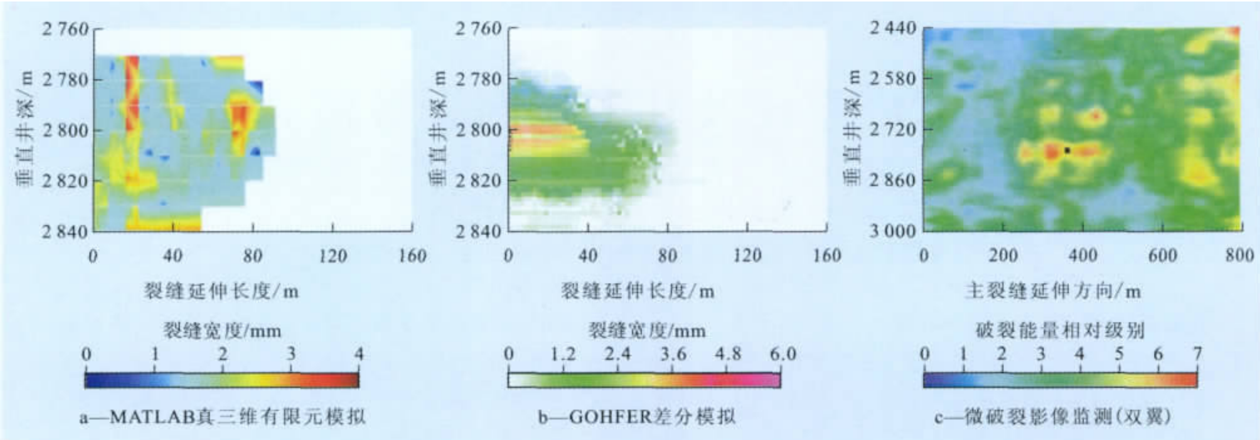

图 1　滨 660 – 斜 24 压裂井裂缝扩展数值模拟结果对比

表 2　滨 660 – 斜 24 压裂井裂缝模拟结果对比

类别	缝高/m	支撑缝长/m	平均缝宽/mm	平均铺砂浓度/(kg · m^{-3})
MATLAB 真三维有限元模拟	72	89	1.89	2.86
GOHFER 差分模拟	59	83	2.67	3.41
微破裂监测	78 ~ 89	80 ~ 120		

6　结束语

对于水力裂缝真三维模型，有限元数值分析比有限差分更为准确，也更切合实际。在 MATLAB 中采用复化辛普森数值积分与其自身开发的符号积分相比，可以使子程序节省 61.9% ~73.5% 的单次运算时间。精确的地层岩石应力强度因子和断裂韧性是准确判断岩石破裂的前提，推导更为符合水力压裂实际的应力强度因子则是改进此程序的下一个重点研究方向。

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application of each method; we also discuss key problems in decline method such as selection of decline mode, original point, decline rate and production unit, determining of stable production period and abandoned point, and the reasons for using of 2-stage-prediction method and decline method. These skills can provide important reference for SEC reserves evaluation and analysis, domestic reserves calculation and recoverable reserves calibration, and provide the proof of oilfield stable production.
Key words: reserve; dynamic evaluation; decline analysis; decline rate; reserve estimation method
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Zhang Shoupeng, Teng Jianbin. Acidification technology and implementation of substep elutriation for low permeability reservoir-case of Xia 463 well in Linnan depression. *PGRE*, 2012, 19(2): 95-97.

Abstract: After stimulation, different well achieves different productivity for the low permeability sandstone reservoir. The key is that the stimulation scheme is not formulated based on the reservoir characteristics. Using the rock sections, cast sections, X diffraction analysis and mineral dissolution experiment techniques, we can accurately understand the characteristics of reservoir rock minerals, and then formulating the corresponding acidification techniques of sub-step elutriation. Using this technique for the low permeability reservoir of Xia 463 well herein, the microcosmic test content and methods for low permeability reservoir are discussed on how to correctly formulate process of acidification technique, as well as the field surveillance. Acidification scheme is proved successful by field operation. It is proved that this technique is not only important for production maintenance, but also for the control of water saturation in low permeability reservoir over a long period of time.
Key words: low permeable sand reservoir; matrix acidification; sub-step elutriation; interparticle material; compatibility
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Wang Shihu, Zhang Zhiang, Wang Lei et al. MATLAB hydraulic fracture propagation simulation technique. *PGRE*, 2012, 19(2): 98-101.

Abstract: Hydraulic fracture simulation technique plays an important role in fracture design and evaluation. It has been developed for many years, however, it is mainly paying more attention to time efficiency and so on, petroleum engineers usually choose 2D or p-3D model for simulation. As we all know, with low accuracy step by step, finite difference method can not satisfy the requirement. What we want to do in this paper is just trying to establish a real 3D hydraulic fracture simulation technique in MATLAB with finite element method. And, then we compare it with the results of the mature commercial software such as GOHFER to improve the accuracy of the simulation and give more reference.
Key words: hydraulic fracture; finite element method; real 3D model; fracture propagation; numerical simulation
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Xue Shifeng, Wang Feifei, Wang Haijing. Numerical study of productivity ratio and factors of perforated well. *PGRE*, 2012, 19(2): 102-105.

Abstract: Perforated completion is widely used, in order to study the effect of perforation factors on perforation productivity ratio and get better perforation process selection, 3D finite element models with factors of perforation parameters (perforation depth, diameter, density and phase), compaction and damage for productivity ratio (PR) calculation in perforation completion are established in this paper. The code connecting software COMSOL and MATLAB are used to simulate and analyze 290 different models with their specific parameters. The effect of perforation depth, diameter, density, phase and compaction on perforation productivity ratio is obtained, and the flow pattern near perforation can be observed by the model. Considering oilfield practice, a simple method of PR calculation and the relationship of well PR and model PR are provided, which will be useful for evaluation of productivity and optimizing perforation completion design.
Key words: perforation completion; productivity ratio; influence factors; finite element model; perforation parameters
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Liu Ming, Zhang Shicheng, Mou Jianye. Dissolution pattern of radial wormhole model in carbonate acidizing. *PGRE*, 2012, 19(2): 106-110.

Abstract: In response to the questions of wormholing during carbonate acidizing, this paper derives a radial two-scale continuum model based on former researches, and studies the dissolving pattern and the effect of some key factors on it, and gets the conditions of the occurrence of wormholes. The results show that: the conclusions got from the models accord well with the experiments conducted by former researchers perfectly; with the rise of diffusion efficiency and decrease of injection rate, the diffusion effect becomes stronger relatively and it is easier to form face dissolution; with the decrease of diffusion efficiency and increase of injection rate, the convection effect becomes more apparent relatively, and it is easier to form uniform dissolution; the wormhole, which can provide adequate permeability and minimize the injection volume of acid at most, is formed when the effect of convection and diffusion is equivalent; the magnitude of heterogeneity has an optimal value, below which the wormhole density and breakthrough volume decrease under more heterogeneity, above which the wormhole density and breakthrough volume become insensitive to the heterogeneity.
Key words: carbonate; acidizing; wormhole; radial model; breakthrough volume; heterogeneity
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